

Problem Set 5

$$① \quad 10,000,000 = \frac{F}{1.06^{20}}$$

$$F = \$32,071,354.72$$

$$② \quad \text{Semi-annual coupon} = 35$$

interest rate = $0.076/2 = 3.8\%$

N	i/y	PV	PMT	FV
4	3.8		35	1,000
- 989.06				

$$\text{Bond Price} = \$989.06$$

$$③ \quad a.$$

N	i/y	PV	PMT	FV
20		- 1,034.74	40	1,000
3.75				

$$\%TM = 2 \times 3.75 = 7.50\%$$

b.

N	1/y	PV	PMT	FV
20	4.5		40	1,000
		- 934.96		

New bond price = \$ 934.96.

④ a. The bond trades at a premium since coupon rate > YTM.

b.

N	1/y	PV	PMT	FV
14	3.5		40	1,000
		- 1,054.60		

The new bond price is \$ 1,054.60.

⑤ a.

N	1/y	PV	PMT	FV
10	6		20	1,000
		- 1,073.60		

When it was issued, the price of the bond was \$ 1,073.60.

b.

N	1/y	PV	PMT	FV
9	6		70	1,000
- 1,068.02				

Bond price after coupon = \$1,068.02

$$\Rightarrow \text{Price before coupon} = 1,068.02 + 70 \\ = \$1,138.02$$

- ⑥ You need to set the coupon rate equal to the YTM of the existing bonds.

N	1/y	PV	PMT	FV
20		-1,078	30	1,000
2.50				

The YTM = $2.50 \times 2 = 5\%$, which should also be the coupon rate so the new bonds are issued at par.

⑦ a.

N	1/y	PV	PMT	FV
60	3		25	1,000
- 861.62				

Bond Price = \$ 861.62.

b.

N	1/y	PV	PMT	FV
59	2.9		25	1,000
- 887.61				

New bond price = \$ 887.61

$$c. \quad HPR = \left(\frac{887.61 + 25}{861.62} \right)^2 - 1 = 12.19\%$$

I'm squaring the gross return to express the rate per year compounded annually.

⑧

Economy	Prob.	YTM	Price	HPR
Boom	0.2	11%	\$74.05	-17.95%
Normal	0.5	8%	\$100.00	8%
Recession	0.3	7%	\$112.28	20.28%

⑨

a. $r_1 = \frac{1,000}{930} - 1 = 7.53\%$

$$r_2 = \left(\frac{1,000}{850} \right)^{1/2} - 1 = 8.47\%$$

$$r_3 = \left(\frac{1,000}{770} \right)^{1/3} - 1 = 9.10\%$$

$$r_4 = \left(\frac{1,000}{700} \right)^{1/4} - 1 = 9.33\%$$

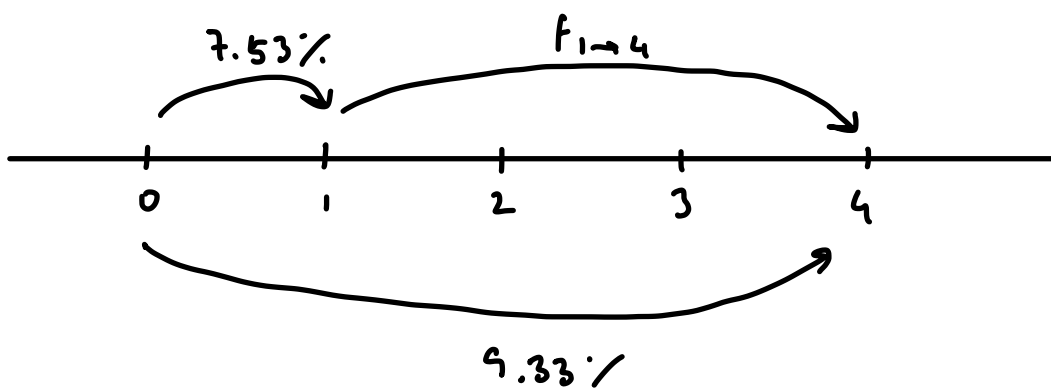
b.

$$B = 0.08 \times 930 + 0.08 \times 850 + 0.08 \times 770 + 1.08 \times 700 = \$960$$

N	1/y	PV	PMT	FV
4		-960	80	1,000
9.24				

$$YTM = 9.24\%$$

c.



$$1.0753 \times (1 + F_{1 \rightarrow 4})^3 = (1.0933)^4$$

$$F_{1 \rightarrow 4} = \left(\frac{(1.0933)^4}{1.0753} \right)^{1/3} - 1 = 9.93\%$$

d. The bond trades for \$985, but it's worth \$960. Sell the bond to receive \$985, and buy 8% of $z(1)$, 8% of $z(2)$, 8% of $z(3)$ and 108% of $z(4)$ which will cost \$960. You make an instant profit of \$25 per trade.

(10) a. $Z(1) = \frac{1,000}{1.10} = \909.09

$$Z(2) = \frac{1,000}{1.09^2} = \$841.68$$

$$Z(3) = \frac{1,000}{1.08^3} = \$793.83$$

b. $1.10 \times (1 + F_{1 \rightarrow 2}) = 1.09^2$

$$F_{1 \rightarrow 2} = \frac{1.09^2}{1.10} - 1 = 8.01\%$$

$$1.09^2 (1 + F_{2 \rightarrow 3}) = 1.08^3$$

$$F_{2 \rightarrow 3} = \frac{1.08^3}{1.09^2} - 1 = 6.03\%$$

$$1.10 \times (1 + F_{1 \rightarrow 3})^2 = 1.08^3$$

$$F_{1 \rightarrow 3} = \left(\frac{1.08^3}{1.10} \right)^{1/2} - 1 = 7.01\%$$

c.

$$B_0 = \frac{85}{1.10} + \frac{85}{1.09^2} + \frac{1,085}{1.08^3} = \$1,010.12$$

d. $B_1 = \frac{85}{1.10} + \frac{1,085}{1.09^2} = \973.97

$$HPR = \frac{973.97 + 85}{1,010.12} - 1 = 4.84\%$$

e. Yes, there is an arbitrage opportunity.

Sell $0.085 Z(1)$, $0.085 Z(2)$ and
 $1.085 Z(3)$ and collect \$1,010.12.

Use \$1,000 to purchase the bond
You make \$10.12 per trade.

(11) a.

T	CF	DCF	Weight
5	10	$\frac{10}{1.08^5} = \$6.81$	$\frac{6.81}{9.29} = 73.26\%$
30	25	$\frac{25}{1.08^{30}} = \$2.4$	$\frac{2.4}{9.29} = 26.74\%$

$$PV = \$9.29$$

$$D = 5 \times 0.7326 + 30 \times 0.2674 = 11.69 \text{ years.}$$

b. Zero-coupon bond with maturity 11.69 years
and face value $9.29 \times (1.08)^{11.69} = \22.84
million.

c. If rates are 9%, the zero coupon
bond is worth

$$\frac{22.84}{1.09^{11.69}} = \$8.34 \text{ million}$$

whereas the liability is worth

$$\frac{10}{1.09^5} + \frac{25}{1.09^{30}} = \$8.38 \text{ million}$$

The net position is

$$8.34 - 8.38 = -\$0.04 \text{ million.}$$

(12) a.

T	CF	DCF	Weight
10	20	$\frac{20}{1.06^{10}} = \$11.168$	47.24%
20	40	$\frac{40}{1.06^{20}} = \$12.472$	52.76%

$$PV = \$23.640$$

$$D = 0.4724 \times 10 + 0.5276 \times 20 = 15.276 \text{ years.}$$

b. You would buy a zero-coupon bond with maturity 15.276 years and face value $23.64 \times 1.06^{15.276} = \57.573 million.

$$c. \quad 5W_5 + 30W_{30} = 5(1 - W_{30}) + 30W_{30} = 15.276$$

$$W_{30} = 41.10\% \quad \text{and} \quad W_5 = 58.90\%$$

Thus, you should invest

$$0.5890 \times 23.64 = \$13.924 \text{ in } B_5$$

$$0.4110 \times 23.64 = \$9.716 \text{ in } B_{30}.$$